



Circles

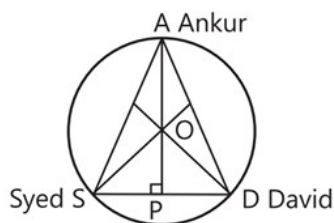
Case Study Based Questions

Case Study 1

A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are standing at equal distance on its boundary each having a toy telephone in his hands to talk each other.



In mathematically; A, S and D are the position of Ankur, Syed and David are sitting at equal distance and O be the position of the centroid of a triangle.



On the basis of the above information, solve the following questions:

Q 1. The length of AP is:

- a. 20 m b. 30 m
- c. 15 m d. 25 m

Q 2. The distance between any two boys is:

- a. $20\sqrt{3}$ m b. $10\sqrt{3}$ m
- c. $15\sqrt{3}$ m d. 20 m

Q 3. The angle between AS and SD is:

- a. 65° b. 60° c. 70° d. 75°

Q 4. $\angle SOD$ is equal to:

- a. 70° b. 60° c. 120° d. 80°

Q 5. $\angle OSP$ is equal to:

- a. 50° b. 60° c. 30° d. 70°

Solutions

1. (b) Let Ankur, Syed and David be represented by A, S and D respectively.

Let $AS = SD = AD = 2a$ m

Since, O is the centroid, so it divides a median in the ratio 2 : 1.

$$\text{So, } \frac{AO}{OP} = \frac{2}{1}$$

$$\Rightarrow \frac{20}{OP} = \frac{2}{1} \quad [\because AO = \text{Radius} = 20 \text{ m}]$$

$$\Rightarrow OP = 10 \text{ m}$$

$$\therefore AP = AO + OP = 20 + 10 = 30 \text{ m}$$

So, option (b) is correct.

2. (a) In right angled $\triangle APS$, by Pythagoras theorem,

$$AS^2 = AP^2 + PS^2$$

$$\Rightarrow (2a)^2 = (30)^2 + a^2 \Rightarrow 4a^2 - a^2 = 900$$

$$\Rightarrow 3a^2 = 900 \Rightarrow a^2 = 300$$

$$\Rightarrow a = 10\sqrt{3} \text{ m}$$

$$\text{So, } 2a = 2 \times 10\sqrt{3} = 20\sqrt{3} \text{ m}$$

Hence, the distance between any two boys is $20\sqrt{3}$ m.

So, option (a) is correct.

3. (b) Since, all three boys are standing at equal distance. Therefore, $\triangle ASD$ is an equilateral triangle.

So, angle between AS and SD is 60° .

So, option (b) is correct.

4. (c) As we know that, the intersection point of altitudes is at the centre of circumcentre of circle. The angle subtended by an arc at the centre is double the angle subtended by it any point on the remaining part of the circle.

$$\therefore \angle SOD = 2 \angle SAD$$

$$= 2 \times 60^\circ$$

$$[\because \angle SAD = \angle ASD = \angle ADS = 60^\circ]$$

$$= 120^\circ$$

So, option (c) is correct.

5. (c) Since, $\angle SOD = 120^\circ$

$$\therefore \angle SOP = \frac{1}{2} \times 120^\circ = 60^\circ \quad [\because AP \text{ is a median}]$$

Now, In $\triangle OSP$,

$$\angle SOP + \angle OSP + \angle OPS = 180^\circ$$

$$\Rightarrow 60^\circ + \angle OSP + 90^\circ = 180^\circ \quad [\because AP \perp SD]$$

$$\Rightarrow \angle OSP = 180^\circ - 150^\circ = 30^\circ$$

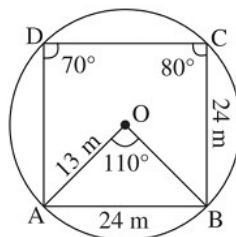
So, option (c) is correct.

Case Study 2

I have a circular garden in the outside of the city. Municipality decided to put some benches, so that the people can sit there and can have some fresh air.



In the given figure, there are two benches of same colour A and B, which are placed at a distance $AB = 24$ m. Similarly, two other benches of same colour C and D are also placed at a same distance of 24 m from each other.



On the basis of the above information, solve the following questions:

Q 1. The measure of $\angle BOC$ is:

- a. 120° b. 55° c. 110° d. 60°

Q 2. The measure of $\angle ABC$ is:

- a. 130° b. 120° c. 105° d. 110°

Q 3. The perpendicular distance from centre O to the chord AB is:

- a. 6 m b. 5 m c. 10 m d. 12 m

Q 4. The value of angle $\angle BAO$ is:

- a. 60° b. 35° c. 70° d. 80°

Q 5. The measure of $\angle DAO$ is:

- a. 70° b. 65° c. 80° d. 85°

Solutions

1. (c) Given, $\angle AOB = 110^\circ$

In the given figure, AB and BC are of equal chords.
Therefore, they subtend equal angles at the centre.

$$\therefore \angle BOC = \angle AOB = 110^\circ$$

So, option (c) is correct.

2. (d) Since, points A, B, C and D lie on the circle.
Therefore, it forms a cyclic quadrilateral.

$$\Rightarrow \angle ABC + \angle ADC = 180^\circ$$

$$\Rightarrow \angle ABC + 70^\circ = 180^\circ$$

$$\therefore \angle ABC = 110^\circ$$

So, option (d) is correct.

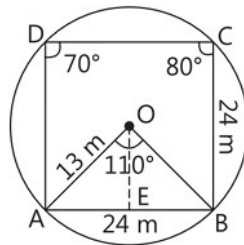
3. (b) Draw a perpendicular line from centre O to the chord AB, which bisects the chord.

$$\therefore AE = BE = 12 \text{ m}$$

In right angle $\triangle OEA$ at E, use Pythagoras theorem

$$\begin{aligned} OE &= \sqrt{(OA)^2 - (AE)^2} \\ &= \sqrt{(13)^2 - (12)^2} = \sqrt{169 - 144} \\ &= \sqrt{25} = 5 \text{ m} \end{aligned}$$

So, option (b) is correct.



4. (b) In $\triangle AOB$,

$$OA = OB$$

[Radii of a circle]

$$\Rightarrow \angle ABO = \angle BAO$$

[Angles opposite to equal sides are equal]

Use angle, sum property of a $\triangle AOB$,

$$\angle ABO + \angle BAO + \angle AOB = 180^\circ$$

$$\Rightarrow \angle BAO + \angle BAO + 110^\circ = 180^\circ$$

$$\Rightarrow 2\angle BAO = 70^\circ$$

$$\therefore \angle BAO = 35^\circ$$

So, option (b) is correct.

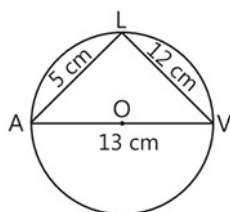
5. (b) Since, A and C are opposite points of a cyclic quadrilateral. Therefore

$$\begin{aligned} & \angle BAD + \angle DCB = 180^\circ \\ \Rightarrow & \angle BAD = 180^\circ - 80^\circ = 100^\circ \\ \Rightarrow & \angle BAO + \angle DAO = 100^\circ \\ \Rightarrow & 35^\circ + \angle DAO = 100^\circ \\ \Rightarrow & \angle DAO = 65^\circ \end{aligned}$$

So, option (b) is correct.

Case Study 3

Three boys Anshul, Vivek and Lalit are playing a game by standing on the boundary of a circle of diameter 13 cm with centre O. Anshul throws a ball to Vivek, Vivek to Lalit and Lalit to Anshul and so on. The distance between Anshul and Lalit is 5 cm and between Vivek and Lalit is 12 cm.



On the basis of the given information, solve the following questions:

Q 1. Measure $\angle ALV$:

- a. 60° b. 90° c. 50° d. 40°

Q 2. The length of the longest chord is:

- a. 12 cm b. 5 cm c. 13 cm d. 16 cm

Q 3. If $\angle LAV = 50^\circ$, then $\angle AVL$ is:

- a. 140° b. 50° c. 40° d. 100°

Q 4. The area covered by these three persons is:

- a. 20 cm^2 b. 30 cm^2 c. 25 cm^2 d. 35 cm^2

Q 5. Equal chords of a circle subtends equal angle at:

- a. centre
b. end point of diameter
c. other than centre
d. None of the above

Solutions

1. (b) Given, AV is a diameter of a circle. So, angle subtended by diameter to the circumference of the circle is 90° .

$$\therefore \angle ALV = 90^\circ$$

So, option (b) is correct.

2. (c) The length of the longest chord in a circle is equal to the length of diameter.

$\therefore$ Length of longest chord = length of diameter

$$= 13 \text{ cm}$$

So, option (c) is correct.

3. (c) In $\triangle AVL$,

$$\angle LAV + \angle AVL + \angle ALV = 180^\circ$$

[by angle sum property of a triangle]

$$\Rightarrow 50^\circ + \angle AVL + 90^\circ = 180^\circ \quad [\because \angle ALV = 90^\circ]$$

$$\therefore \angle AVL = 40^\circ$$

So, option (c) is correct.

4. (b) The area covered by these three persons is equal to the area of $\triangle AVL$.

$$\therefore \text{Area of } \triangle AVL = \frac{1}{2} \times AL \times LV$$

$$= \frac{1}{2} \times 5 \times 12 = 30 \text{ cm}^2$$

So, option (b) is correct.

5. (a) Equal chords of a circle subtends equal angle at the centre.

So, option (a) is correct.